STPM/S(E)956

PEPERIKSAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(*MALAYSIA HIGHER SCHOOL CERTIFICATE*)

FURTHER MATHEMATICS T
Syllabus and Specimen Papers

This syllabus applies for the 2002 examination and thereafter until further notice. Teachers/candidates are advised to contact Majlis Peperiksaan Malaysia for the latest information about the syllabus.

MPM

MAJLIS PEPERIKSAAN MALAYSIA
(*MALAYSIAN EXAMINATIONS COUNCIL*)

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FALSAFAH PENDIDIKAN KEBANGSAAN

"Pendidikan di Malaysia adalah suatu usaha berterusan ke arah memperkembangkan lagi potensi individu secara menyeluruh dan bersepadu untuk mewujudkan insan yang seimbang dan harmonis dari segi intelek, rohani, emosi, dan jasmani berdasarkan kepercayaan dan kepatuhan kepada Tuhan. Usaha ini adalah bagi melahirkan rakyat Malaysia yang berilmu pengetahuan, berketrampilan, berakhlak mulia, bertanggungjawab, dan berkeupayaan mencapai kesejahteraan diri serta memberi sumbangan terhadap keharmonian dan kemakmuran masyarakat dan negara".

FOREWORD

Mathematics is a diverse and growing field of study. The study of mathematics can contribute towards clear, logical, quantitative and relational thinking, and also facilitates the implementation of programmes which require mathematical modeling, statistical analysis, and computer technology. Mathematics is finding ever wider areas of applications.

The mathematics syllabus for the Malaysia Higher School Certificate examination has been reviewed and rewritten so as to be more relevant to the current needs. The Further Mathematics T syllabus aims to extend the command of mathematical concepts and skills, as well as the interest in mathematics and its applications, for students who intend to proceed to programmes related to science and technology at institutions of higher learning. The aims, objective, contents, form of examination, reference books, and specimen papers are presented in this booklet.

On behalf of Malaysian Examinations Council, I would like to thank the Malaysia Higher School Certificate Examination Mathematics Syllabus Committee chaired by Associate Professor Dr Harun bin Budin and all others who have contributed towards the development of this syllabus. It is hope that this syllabus will achieve its aims.

DATO' HAJI TERMUZI BIN HAJI ABDUL AZIZ
Chief Executive
Malaysian Examinations Council

CONTENTS

SYLLABUS

956 FURTHER MATHEMATICS T

(May only be taken together with 954 Mathematics T)

Aims

The Further Mathematics T syllabus aims to extend the command of mathematical concepts and skills, as well as the interest in mathematics and its applications, for students who intend to proceed to programmes related to science and technology at institutions of higher learning.

Objectives

The objectives of this syllabus are to develop the abilities of students to

(*a*) understand and use mathematical terminology, notations, principles, and methods;

(*b*) perform calculations accurately and carry out appropriate estimations and approximations;

(*c*) understand and use information in tabular, diagrammatic, and graphical forms;

(*d*) analyse and interpret data;

(*e*) formulate problems into mathematical terms and solve them;

(*f*) interpret mathematical results and make inferences;

(*g*) present mathematical arguments in a logical and systematic manner.

Content

1. Logic and proof

1.1 Logic

1.2 Proof

Explanatory notes

Candidates should be able to

(*a*) understand propositions and quantifiers;

(*b*) use negations, conjunctions, disjunctions, implications, and equivalences, together with their symbols;

(*c*) use the converse and the contrapositive of an implication;

(*d*) suggest a counter-example to negate a proposition;

(*e*) use direct and indirect proofs;

(*f*) suggest a general result by induction based on a certain number of examples or trials;

(*g*) use the method of mathematical induction to prove a result.

2. Complex numbers

2.1 Polar form

2.2 de Moivre's theorem

2.3 Equations

Explanatory notes

Candidates should be able to

(*a*) express a complex number in polar form;

(*b*) carry out multiplication and division of complex numbers expressed in polar form;

(*c*) understand the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying, and dividing two complex numbers;

(*d*) prove de Moivre's theorem for positive integer exponents;

(*e*) use de Moivre's theorem;

(*f*) find roots of unity;

(*g*) solve equations involving complex numbers;

(*h*) illustrate equations involving complex numbers as loci in the Argand diagram.

3. Matrices

3.1 Row and column operations

3.2 System of linear equations

3.3 Eigenvalues and eigenvectors

Explanatory notes

Candidates should be able to

(*a*) carry out row and column operations on a matrix;

(*b*) use the properties of the determinant of a matrix;

(*c*) find the inverse of a non-singular 3×3 matrix;

(*d*) understand the consistency of a system of equations and the uniqueness of its solution;

(*e*) solve simultaneous linear equations by using the Gaussian elimination;

(*f*) use Cramer's rule for solving simultaneous linear equations;

(*g*) find and use the eigenvalues and eigenvectors of a matrix (restricted to real eigenvalues and eigenvectors);

(*h*) use Cayley-Hamilton theorem.

4. Recurrence relations

4.1 Recurrence relations

4.2 Homogeneous linear recurrence relations

4.3 Non-homogeneous linear recurrence relations

Explanatory notes

Candidates should be able to

(*a*) understand the concept of a recurrence relation and its order;

(*b*) find the general solution of a first or second order homogeneous linear recurrence relation with constant coefficients;

(*c*) find the general solution of a first or second order non-homogeneous linear recurrence relation with constant coefficients;

(*d*) use the boundary conditions to find a particular solution;

(*e*) solve problems that can be modelled by recurrence relations.

5. Functions

5.1 Inverse trigonometric functions

5.2 Hyperbolic functions

5.3 Inverse hyperbolic functions

Explanatory notes

Candidates should be able to

(*a*) use the functions $\sin^{-1}x$, $\cos^{-1}x$, and $\tan^{-1}x$, and their graphs;

(*b*) solve equations involving inverse trigonometric functions;

(*c*) use the six hyperbolic functions and their graphs;

(*d*) use the formulae $\cosh^2 x - \sinh^2 x = 1$, $1 - \tanh^2 x = \text{sech}^2 x$, $\coth^2 x - 1 = \text{cosech}^2 x$;

(*e*) derive and use the formulae for $\sinh(x \pm y)$, $\cosh(x \pm y)$, $\tanh(x \pm y)$;

(*f*) solve equations involving hyperbolic functions;

(*g*) use the functions $\sinh^{-1}x$, $\cosh^{-1}x$, and $\tanh^{-1}x$, and their graphs;

(*h*) derive and use the logarithmic forms for $\sinh^{-1}x$, $\cosh^{-1}x$, $\tanh^{-1}x$;

(*i*) solve equations involving inverse hyperbolic functions.

6. Differentiation and integration

6.1 Differentiability of a function

6.2 Derivatives of a function defined implicitly or parametrically

6.3 Derivatives and integrals of trigonometric and inverse trigonometric functions

6.4 Derivatives and integrals of hyperbolic and inverse hyperbolic functions

6.5 Reduction formulae

6.6 Applications of integration

Explanatory notes

Candidates should be able to

(*a*) determine the differentiability of a function;

(*b*) use the relationship between the differentiability and continuity of a function;

(*c*) find the second derivative of an implicit function;

(*d*) find the second derivative of a function defined parametrically;

(*e*) derive and use the derivatives of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1}x$, $\cosh^{-1}x$, $\tanh^{-1}x$;

(*f*) use the integrals of $\dfrac{1}{a^2+x^2}$, $\dfrac{1}{\sqrt{a^2-x^2}}$, $\dfrac{1}{\sqrt{x^2+a^2}}$, $\dfrac{1}{\sqrt{x^2-a^2}}$;

(*g*) differentiate and integrate functions involving trigonometric and inverse trigonometric functions;

(*h*) differentiate and integrate functions involving hyperbolic and inverse hyperbolic functions;

(*i*) use trigonometric and hyperbolic substitutions for integration;

(*j*) prove and use reduction formulae for the evaluation of definite integrals;

(*k*) calculate the length of an arc of a curve whose equation is expressed in cartesian coordinates or in terms of a parameter;

(*l*) calculate the surface area of revolution when an arc of a curve, whose equation is expressed in cartesian coordinates or in terms of a parameter, is rotated about one of the coordinate axes.

7. Power series

7.1 Taylor polynomials

7.2 Taylor series

Explanatory notes

Candidates should be able to

(*a*) know that a certain function can be approximated by a Taylor polynomial with the remainder term $R_n(a;x) = \displaystyle\int_a^x \frac{(x-t)^n}{n!}\mathrm{f}^{\,n+1}(t)\,\mathrm{d}t$;

(*b*) use the remainder term of the form $\frac{(x-a)^{n+1}}{(n+1)!}\mathrm{f}^{\,n+1}(c)$, where $c \in (a, x)$;

(*c*) derive and use the Taylor or Maclaurin series of a given function;

(*d*) use differentiation and integration of a series to obtain certain power series;

(*e*) use the expansion of a series to determine the limit of a function.

8. Differential equations

8.1 First order linear differential equations

8.2 Second order linear differential equations

Explanatory notes

Candidates should be able to

(*a*) find the general solution of a first order linear differential equation by using an integrating factor;

(*b*) find the complementary function and a particular integral of a second order linear differential equation with constant coefficients;

(*c*) know that the general solution of a second order linear differential equation with constant coefficients is the sum of the complementary function and a particular integral;

(*d*) find the general solution of a differential equation that can be transformed into a first or second order linear differential equation with constant coefficients;

(*e*) use the boundary conditions to find a particular solution;

(*f*) solve problems that can be modelled by differential equations.

9. Number theory

9.1 Divisibility

9.2 Modular arithmetic

Explanatory notes

Candidates should be able to

(*a*) use the divisibility properties of integers;

(*b*) use the divisibility criteria for determining the divisibility of a number by a small positive integer;

(*c*) understand prime numbers, composite numbers, and unique factorisation;

(*d*) find greatest common divisors and least common multiples;

(*e*) use Euclid’s algorithm;

(*f*) use the properties of congruence $a \equiv b \pmod{n}$;

(*g*) carry out addition, subtraction, and multiplication of integers modulo n;

(*h*) use the properties of integers modulo n;

(*i*) solve simultaneous linear congruences;

(*j*) use congruences to determine the divisibility of an integer.

10. Graph theory

10.1 Graphs

10.2 Paths and cycles

10.3 Matrix representations

Explanatory notes

Candidates should be able to

(*a*) understand the concept of a graph and its representation in a diagram;

(*b*) use the relationship between the sum of the degrees of vertices and the number of edges;

(*c*) understand the meaning of simple graphs, complete graphs, and bipartite graphs;

(*d*) understand the terms walk, trail, path, circuit, and cycle;

(*e*) understand the idea of a connected graph;

(*f*) determine eulerian trails and circuits;

(*g*) detemine hamiltonian paths and cycles;

(*h*) represent a graph by an adjacency or incidence matrix;

(*i*) solve problems that can be modelled by graphs.

11. Transformation geometry

11.1 Transformations

11.2 Matrix representations

Explanatory notes

Candidates should be able to

(*a*) understand transformation as a correspondence between two sets of points in a plane;

(*b*) use the properties of isometries of the plane: rotation, translation, reflection;

(*c*) use the properties of similarity transformations in two dimensions;

(*d*) use the properties of stretches and shears in two dimensions;

(*e*) describe a linear transformation;

(*f*) use a 2×2 matrix to represent a linear transformation;

(*g*) find the image under a linear transformation;

(*h*) use the relationship between the area scale-factor of a linear transformation and the determinant of the corresponding matrix;

(*i*) carry out addition, subtraction, scalar multiplication, and composition of linear transformations.

12. Coordinate geometry

12.1 Three-dimensional vectors

12.2 Straight lines

12.3 Planes

Explanatory notes

Candidates should be able to

(*a*) understand the concept of three-dimensional vectors;

(*b*) carry out operations on vectors, including scalar and vector products;

(*c*) use the properties of vectors;

(*d*) use the equation of a line in either of the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ or $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$;

(*e*) find the equation of a plane in any of the forms $(\mathbf{r} - \mathbf{a}) \bullet \mathbf{n} = 0$, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, or $ax + by + cz = d$;

(*f*) determine whether two lines are skew, are parallel, or intersect;

(*g*) find the point of intersection of two lines;

(*h*) determine whether a line lies in a plane, is parallel to a plane, or intersects a plane;

(*i*) find the point of intersection of a line and a plane;

(*j*) find the line of intersection of two non-parallel planes;

(*k*) calculate the distance from a point to a line or to a plane;

(*l*) calculate the angle between two lines, between a line and a plane, or between two planes.

13. Sampling and estimation

13.1 Random samples

13.2 Sampling distributions

13.3 Point estimates

13.4 Interval estimates

Explanatory notes

Candidates should be able to

(*a*) understand the distinction between a population and a random sample and between a parameter and a statistic;

(*b*) use the sampling distributions of the sample proportion and mean;

(*c*) use the central limit theorem;

(*d*) calculate unbiased estimates for the population proportion, mean, and variance;

(*e*) use *t*-distribution tables;

(*f*) calculate and interpret the standard error of the estimator for the population proportion or mean;

(*g*) obtain and interpret a confidence interval for the population proportion;

(*h*) obtain and interpret a confidence interval for the population mean based on a large sample;

(*i*) obtain and interpret a confidence interval for the population mean based on a small sample from a normal distribution;

(*j*) determine the sample size in the estimation of the population proportion or mean.

14. Hypothesis testing

14.1 Hypotheses

14.2 Critical regions

14.3 Tests of significance

Explanatory notes

Candidates should be able to

(*a*) understand and formulate null and alternative hypotheses;

(*b*) understand and determine test statistics;

(*c*) understand the concept of a significance level;

(*d*) determine critical regions;

(*e*) carry out a hypothesis test concerning the population proportion based on a sample drawn from a binomial distribution;

(*f*) carry out a hypothesis test concerning the population mean based on a large sample;

(*g*) carry out a hypothesis test concerning the population mean based on a small sample drawn from a normal distribution;

(*h*) understand and find type I and type II errors.

15. χ^2 tests

15.1 χ^2 distribution

15.2 Tests for goodness of fit

15.3 Tests for independence

Explanatory notes

Candidates should be able to

(*a*) understand the χ^2 distribution;

(*b*) use χ^2 distribution tables;

(*c*) use a χ^2 test to determine goodness of fit;

(*d*) use a χ^2 test to determine independence in a contingency table.

16. Correlation and regression

16.1 Scatter diagrams

16.2 Pearson correlation coefficient

16.3 Linear regression lines

Explanatory notes

Candidates should be able to

(*a*) understand the concept of correlation and the relationship between the value of a correlation coefficient and the scatter diagram;

(*b*) calculate and interpret a Pearson correlation coefficient;

(*c*) understand the concept of linear regression;

(*d*) find the equation of a linear regression line by the method of least squares (transformation of variables may be required to obtain a linear relation);

(*e*) draw a regression line on the scatter diagram;

(*f*) use a suitable regression line for making predictions and understand the uncertainties of such predictions;

(*g*) use the relationships between correlation coefficient, regression coefficient, and coefficient of determination.

Form of Examination

The examination consists of two papers; the duration for each paper is 3 hours. Candidates are required to take both Paper 1 and Paper 2.

Paper 1 is based on topics 1 to 8 and Paper 2 is based on topics 9 to 16. Each paper contains 12 compulsory questions of variable mark allocations totalling 100 marks.

Reference Books

1. Bostock, L., Chandler, S., & Rourke, C., *Further Pure Mathematics*, Stanley Thornes, 1982 (Reprinted 2002).

2. Gaulter, B. & Gaulter, M., *Further Pure Mathematics*, Oxford, 2001.

3. Poole, D., *Linear Algebra: A Modern Introduction*, Brooks/Cole, 2003.

4. Anton, H., Bivens, I., & Davis, S., *Calculus* (Seventh Edition), John Wiley, 2002.

5. Barnett, S., *Discrete Mathematics: Numbers and Beyond*, Addison Wesley, 1998.

6. Rosen, K. H., *Discrete Mathematics and Its Applications* (Fifth Edition), McGraw-Hill, 2003.

7. Scheinerman, E. R., *Mathematics: A Discrete Introduction*, Brooks/Cole, 2000.

8. Crawshaw, J. & Chambers, J., *A concise Course in Advanced Level Statistics* (Fourth Edition), Nelson Thornes, 2001.

9. Johnson, R. A. & Bhattacharyya, G. K., *Statistics: Principles and Methods* (Fourth Edition), John Wiley, 2001.

10. Upton, G. & Cook, I., *Introducing Statistics* (Second Edition), Oxford, 2001.

SPECIMEN PAPER

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FURTHER MATHEMATICS T

PAPER 1

(Three hours)

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Instructions to candidates:

Answer **all** *questions.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae, and graph paper are provided.

1 Let $n \in \mathrm{N}$. Show that n^2 is odd if n is odd. Deduce that n is even if n^2 is even. [4 *marks*]

2 Show that, for all values of x in the interval $-1 \le x \le 1$,

$$\sin^{-1} x + \cos^{-1} x = \tfrac{1}{2}\pi.$$ [4 *marks*]

3 Given that 7 sinh x + 20 cosh x = 24, find all possible values of tanh = $\tfrac{1}{2}x$. [4 *marks*]

4 Solve the equation $z^6 - z^3 + 1 = 0$, giving your answers in the form $re^{i\theta}$. [6 *marks*]

5 Determine the degree of a Taylor polynomial suitable for estimating the value of sin 0.5 so that the error of approximation does not exceed $\tfrac{1}{2}(10^{-3})$. Find sin 0.5 correct to three decimal places.

[6 *marks*]

6 The point P in an Argand diagram represents the complex number z. If $z = \dfrac{4}{2 + \mathrm{i}t}$, where t is a non-negative real parameter, find the cartesian equation of the locus of P and sketch the locus.

[7 *marks*]

7 Given that $a_1 = 2p$ where $p \ne 0$ and $a_n = 2p - \dfrac{p^2}{a_{n-1}}$ for $n > 1$, show, by mathematical induction, that $a_n \ne p$ for all $n \ge 1$. [5 *marks*]

Hence show that the sequence $\dfrac{1}{a_1 - p}, \dfrac{1}{a_2 - p}, \cdots, \dfrac{1}{a_n - p}$ is an arithmetic progression.

[4 *marks*]

8 Solve the recurrence relation

$$u_n = 4u_{n-1} - 2u_{n-2}, \text{ with } u_0 = -1,\ u_1 = 0.$$

Show that the solution may be written in the form

$$u_n = p(2 + \sqrt{2})^{n-1} + q(2 - \sqrt{2})^{n-1},$$

where p and q are constants to be determined. [7 *marks*]

Deduce that, for $n \ge 2$, u_n is the integer nearest to $\dfrac{1}{\sqrt{2}}(2 + \sqrt{2})^{n-1}$. [2 *marks*]

9 Given that $I_n = \displaystyle\int_0^1 \frac{1}{(1 + x^2)^n}\,\mathrm{d}x$, show that, for $n \ge 1$,

$$2n I_{n+1} = (2n - 1)I_n + \frac{1}{2^n}.$$ [5 *marks*]

Hence find the value of $\displaystyle\int_0^1 \frac{1}{(1 + x^2)^4}\,\mathrm{d}x$. [4 *marks*]

10 By using the substitution $x = e^z$, show that the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0,$$

where $x > 0$, can be transformed into

$$\frac{d^2 y}{dz^2} + y = 0.$$ [6 *marks*]

Hence find y in terms of x subject to the condition that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 1$.

[6 *marks*]

11 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 2 & 0 \end{pmatrix}.$$

(i) By using the Cayley Hamilton theorem, find $\mathbf{A}^{-1}$. [3 *marks*]

(ii) Find $\mathbf{A}^8 - \mathbf{3A}^6 + 2\mathbf{A}^5 + 2\mathbf{A}^4 - \mathbf{6A}^2 + 5\mathbf{A} + 2\mathbf{I}$, where **I** is the 3×3 identity matrix.

[3 *marks*]

(iii) Find the eigenvalues and corresponding eigenvectors of the matrix **A**. Write down an eigenvalue of $\mathbf{A}^8$ and the corresponding eigenvector. [9 *marks*]

12 Prove that

$$\sinh^{-1} x = \ln(x + \sqrt{1 + x^2}).$$ [3 *marks*]

Show that

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}},$$

and find the second and third derivatives of $\sinh^{-1} x$. [3 *marks*]

Obtain the Maclaurin expansion for $\sinh^{-1} x$ as far as the term in x^3. [3 *marks*]

Sketch the curve $y = \sinh^{-1} x$. State the maximum gradient of the curve and hence write down the range of values of k for which the equation

$$\sinh^{-1} x = kx$$

has three distinct real roots. [4 *marks*]

By using the expansion you obtain, estimate the positive root of the equation

$$\sinh^{-1} x = \frac{53}{54} x.$$ [2 *marks*]

SPECIMEN PAPER

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FURTHER MATHEMATICS T

PAPER 2

(Three hours)

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Instructions to candidates:

Answer **all** *questions.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae, and graph paper are provided.

1 If the line segments AB and $A'B'$ are equal in length, describe the reflections P and Q such that QP transforms A into A' and B into B'. [4 *marks*]

2 If r and s are positive integers and a is an integer with $a \geq 2$, show that $a^r - 1$ divides $a^{rs} - 1$.

Hence show that, if $a^n - 1$ is prime and n is an integer with $n \geq 2$, then n is prime and $a = 2$.
[6 *marks*]

3 Use the Euclid's algorithm to find the greatest common divisor of 12345 and 54322.
[3 *marks*]

Hence find the smallest positive integer n such that

$$(12345)\, n \equiv 1 \bmod 54322,$$

and further solve the equation

$$(12345)\, n \equiv 3 \bmod 54322.$$ [5 *marks*]

4 The straight line l passes through the point A with position vector $\mathbf{a}$ and is parallel to the unit vector $\mathbf{e}$. The point R lies on the line l and has position vector $\mathbf{r}$. Show that

$$(\mathbf{r} - \mathbf{a}) \times \mathbf{e} = \mathbf{0}.$$ [2 *marks*]

The point P has position vector $\mathbf{p}$. Show that the perpendicular distance from P to the line l is $|(\mathbf{p} - \mathbf{a}) \times \mathbf{e}|$. [2 *marks*]

Find the perpendicular distance from the origin to the straight line that passes through the points (5, 1, –2) and (2, –1, 4). [4 *marks*]

5 A transformation T of the x-y plane is defined by

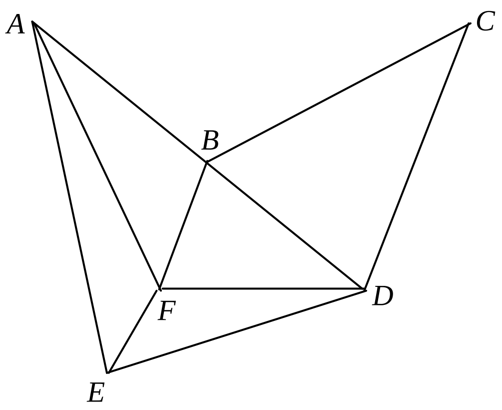
$$\mathrm{T} : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \mathbf{M} \begin{pmatrix} x \\ y \end{pmatrix},$$

where $\mathbf{M}$ is a 2×2 matrix. Under T, the points (1, 2) and (–3, 4) are transformed into the points (4, 3) and (–2, 11) respectively. Determine the matrix $\mathbf{M}$. [5 *marks*]

(i) Show that T transforms a straight line into a straight line. [4 *marks*]

(ii) The circle $x^2 + y^2 = 9$ is transformed into the circle C under T. Find the area of the circle C.
[3 *marks*]

6



Write down the adjacency matrix **M** for the above graph based on the sequence of vertices *A*, *B*, *C*, *D*, *E*, and *F*. State what the sum of the entries in each row of **M** represents. [3 *marks*]

Comment on this graph based on the entries in $\mathbf{M}^2$. [4 *marks*]

(i) If the above graph represents a system of roads connecting six towns *A*, *B*, *C*, *D*, *E*, and *F*, and the following table shows the bus fares between these towns, determine a route from *A* to *D* with the lowest fare. [3 *marks*]

Fares between towns (RM)

	A	*B*	*C*	*D*	*E*	*F*
A	–	6	–	–	4	5
B	6	–	3	7	–	4
C	–	3	–	3	–	–
D	–	7	3	–	5	8
E	4	–	–	5	–	6
F	5	4	–	8	6	–

(ii) If the above graph represents the roads which may be constructed to connect six villages *A*, *B*, *C*, *D*, *E*, and *F*, and the following table shows the cost of constructing the roads connecting these villages, determine the network of roads which should be constructed so that each village is connected to all the others directly or indirectly and the total cost is minimum. Find the minimum cost. [5 *marks*]

Costs of constructing the roads between villages (RM million)

	A	*B*	*C*	*D*	*E*	*F*
A	–	3	–	–	5	7
B	3	–	9	6	–	5
C	–	9	–	8	–	–
D	–	6	8	–	7	9
E	5	–	–	7	–	4
F	7	5	–	9	4	–

7 The correlation coefficient between the variables p and q for 10 pairs of values is -0.5. State whether it is true or false that on the average p increases twice as fast as q decreases. Justify your answer. [3 *marks*]

8 In a sample of 50 moths from the National Park, there are 27 female moths. Obtain a 95% confidence interval for the proportion of female moths in the National Park. [5 *marks*]

9 A normal population has a mean of 75 and a variance of 25. Obtain the smallest sample size needed so that the probability that the error of estimation is less than 1 is at least 0.90. [5 *marks*]

10 A car manufacturing company claims that the colours preferred by its customers are white, grey, blue, and red in the ratio 7 : 3 : 3 : 1. It is found that, from 420 prospective buyers listed in the company's record of order, 225 persons choose white, 90 grey, 85 blue, and 20 red. Based on this record of orders, test whether the claim made by the company can be accepted at the 5% significance level. [6 *marks*]

11 A marmalade manufacturer produces thousands of jars of marmalade each week. The mass of marmalade in a jar is an observation from a normal distribution having mean 455 g and standard deviation 0.8 g

Following a slight adjustment to the filling machine, a random sample of 10 jars is found to contain the following masses (in g) of marmalade:

454.8, 453.8, 455.0, 454.4, 455.4, 454.4, 454.4, 455.0, 455.0, 453.6.

(i) Assuming that the variance of the distribution is unaltered by the adjustment, test, at the 5% significance level, the hypothesis that there has been no change in the mean of the distribution.

[5 *marks*]

(ii) Assuming that the variance of the distribution may have been altered, obtain an unbiased estimate of the new variance and, using this estimate, test, at the 5% significance level, the hypothesis that there has been no change in the mean of the distribution. [7 *marks*]

12 The following table shows the price y, in RM, and the age x, in years, of six used cars of a certain model in Kuantan.

x	2	4	4	6	6	8
y	56 800	54 600	54 500	52 500	52 000	50 000

(i) Plot the scatter diagram of $\log_{10} y$ against x. [3 *marks*]

(ii) Taking $\log_{10} y = a + bx$ as the equation of the regression line of $\log_{10} y$ on x, find the least squares estimates for a and b correct to three decimal places. [7 *marks*]

(iii) Draw this regression line on your scatter diagram. [2 *marks*]

(iv) Estimate the price of a used car of the same model in Kuantan which is 5 years old. [2 *marks*]

(v) State whether it is appropriate to use this regression line to predict the price of a used car of the same model in Kuantan which is 12 years old. Give a reason for your answer.

[2 *marks*]