STPM/S(E)950

PEPERIKSAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(*MALAYSIA HIGHER SCHOOL CERTIFICATE*)

MATHEMATICS S
Syllabus and Specimen Papers

This syllabus applies for the 2002 examination and thereafter until further notice. Teachers/candidates are advised to contact Majlis Peperiksaan Malaysia for the latest information about the syllabus.

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(*MALAYSIAN EXAMINATIONS COUNCIL*)

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FALSAFAH PENDIDIKAN KEBANGSAAN

"Pendidikan di Malaysia adalah suatu usaha berterusan ke arah memperkembangkan lagi potensi individu secara menyeluruh dan bersepadu untuk mewujudkan insan yang seimbang dan harmonis dari segi intelek, rohani, emosi, dan jasmani berdasarkan kepercayaan dan kepatuhan kepada Tuhan. Usaha ini adalah bagi melahirkan rakyat Malaysia yang berilmu pengetahuan, berketrampilan, berakhlak mulia, bertanggungjawab, dan berkeupayaan mencapai kesejahteraan diri serta memberi sumbangan terhadap keharmonian dan kemakmuran masyarakat dan negara".

FOREWORD

Mathematics is a diverse and growing field of study. The study of mathematics can contribute towards clear, logical, quantitative and relational thinking, and also facilitates the implementation of programmes which require mathematical modeling, statistical analysis, and computer technology. Mathematics is finding ever wider areas of applications.

The mathematics syllabus for the Malaysia Higher School Certificate examination has been reviewed and rewritten so as to be more relevant to the current needs. The Mathematics S syllabus aims to develop the understanding of mathematical concepts and their applications, together with the skills in mathematical reasoning and problem solving, so as to enable students to proceed to programmes related to social sciences and management at institutions of higher learning. The aims, objective, contents, form of examination, reference books, and specimen papers are presented in this booklet.

On behalf of Malaysian Examinations Council, I would like to thank the Malaysia Higher School Certificate Examination Mathematics Syllabus Committee chaired by Associate Professor Dr Harun bin Budin and all others who have contributed towards the development of this syllabus. It is hope that this syllabus will achieve its aims.

DATO' HAJI TERMUZI BIN HAJI ABDUL AZIZ
Chief Executive
Malaysian Examinations Council

CONTENTS

SYLLABUS

950 MATHEMATICS S

(May not be taken with 954 Mathematics T or 956 Further Mathematics T)

Aims

The Mathematics S syllabus aims to develop the understanding of mathematical concepts and their applications, together with the skills in mathematical reasoning and problem solving, so as to enable students to proceed to programmes related to social sciences and management at institutions of higher learning.

Objectives

The objectives of this syllabus are to develop the abilities of students to

(*a*) understand and use mathematical terminology, notation, principles, and methods;

(*b*) perform calculations accurately and carry out appropriate estimations and approximations;

(*c*) understand and use information in tabular, diagrammatic, and graphical forms;

(*d*) analyse and interpret data;

(*e*) formulate problems into mathematical terms and solve them;

(*f*) interpret mathematical results and make inferences;

(*g*) present mathematical arguments in a logical and systematic manner.

Content

1. Numbers and sets

1.1 Real numbers

1.2 Exponents and logarithms

1.3 Complex numbers

1.4 Sets

Explanatory notes

Candidates should be able to

(*a*) understand the real number system;

(*b*) carry out elementary operations on real numbers;

(*c*) use the properties of real numbers;

(*d*) use the notation for intervals of real numbers;

(*e*) use the notation $|x|$ and its properties;

(*f*) understand integral and rational exponents;

(*g*) understand the relationship between logarithms and exponents;

(*h*) carry out change of base for logarithms;

(*i*) use the laws of exponents and laws of logarithms;

(*j*) use the results: for $a > b$ and $c > 1$, $c^a > c^b$ and $\log_c a > \log_c b$; for $a > b$ and $0 < c < 1$, $c^a < c^b$ and $\log_c a < \log_c b$;

(*k*) solve equations and inequalities involving exponents and logarithms;

(*l*) understand the meaning of the real part, imaginary part, and conjugate of a complex number;

(*m*) find the modulus and argument of a complex number;

(*n*) represent complex numbers geometrically by means of an Argand diagram;

(*o*) use the condition for the equality of two complex numbers;

(*p*) carry out elementary operations on complex numbers expressed in cartesian form;

(*q*) understand the concept of a set and set notation;

(*r*) carry out operations on sets;

(*s*) use the laws of the algebra of sets.

2. Polynomials

2.1 Polynomials

2.2 Equations and inequalities

2.3 Partial fractions

Explanatory notes

Candidates should be able to

(*a*) understand the meaning of the degrees and coefficients of polynomials;

(*b*) carry out elementary operations on polynomials;

(*c*) use the condition for the equality of two polynomials;

(*d*) find the factors and zeroes of polynomials;

(*e*) prove and use the remainder and factor theorems;

(*f*) use the process of completing the square for a quadratic polynomial;

(*g*) derive the quadratic formula;

(*h*) solve linear, quadratic, and cubic equations and equations that can be transformed into quadratic or cubic equations;

(*i*) use the discriminant of a quadratic equation to determine the properties of its roots;

(*j*) prove and use the relationships between the roots and coefficients of a quadratic equation;

(*k*) solve inequalities involving polynomials of degrees not exceeding three, rational functions, and the modulus sign;

(*l*) solve a pair of simultaneous equations involving polynomials of degrees not exceeding three;

(*m*) express rational functions in partial fractions.

3. Sequences and series

3.1 Sequences

3.2 Series

3.3 Binomial expansions

Explanatory notes

Candidates should be able to

(*a*) use an explicit or a recursive formula for a sequence to find successive terms;

(*b*) determine whether a sequence is convergent or divergent and find the limit of a convergent sequence;

(*c*) use the Σ notation;

(*d*) use the formula for the general term of an arithmetic or a geometric progression;

(*e*) derive and use the formula for the sum of the first n terms of an arithmetic or a geometric series;

(*f*) use the formula for the sum to infinity of a convergent geometric series;

(*g*) solve problems involving arithmetic or geometric progressions or series;

(*h*) use the method of differences to obtain the sum of a finite or a convergent infinite series;

(*i*) expand $(a + b)^n$ where n is a positive integer;

(*j*) expand $(1 + x)^n$ where n is a rational number and $|x| < 1$;

(*k*) use the binomial expansion for approximation.

4. Matrices

4.1 Matrices

4.2 Inverse matrices

4.3 System of linear equations

Explanatory notes

Candidates should be able to

(*a*) understand the terms null matrix, identity matrix, diagonal matrix, and symmetric matrix;

(*b*) use the condition for the equality of two matrices;

(*c*) carry out matrix addition, matrix subtraction, scalar multiplication, and matrix multiplication for matrices with at most three rows and three columns;

(*d*) find the minors, cofactors, determinants, and adjoints of 2×2 and 3×3 matrices;

(*e*) find the inverses of 2×2 and 3×3 non-singular matrices;

(*f*) use the result, for non-singular matrices, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$;

(*g*) use inverse matrices for solving simultaneous linear equations;

(*h*) solve problems involving the use of a matrix equation.

5. Coordinate geometry

5.1 Cartesian coordinates in a plane

5.2 Straight lines

5.3 Curves

Explanatory notes

Candidates should be able to

(*a*) understand cartesian coordinates for the plane and the relationship between a graph and an associated algebraic equation;

(*b*) calculate the distance between two points and the gradient of the line segment joining two points;

(*c*) find the coordinates of the mid-point and the point that divides a line segment in a given ratio;

(*d*) find the equation of a straight line;

(*e*) use the relationships between gradients of parallel lines and between gradients of perpendicular lines;

(*f*) calculate the distance from a point to a line;

(*g*) determine the equation of a circle and identify its centre and radius;

(*h*) use the equations and graphs of ellipses, parabolas, and hyperbolas;

(*i*) use the parametric representation of a curve (excluding trigonometric expressions);

(*j*) find the coordinates of a point of intersection;

(*k*) solve problems concerning loci.

6. Functions

6.1 Functions and graphs

6.2 Composite functions

6.3 Inverse functions

6.4 Limit and continuity of a function

Explanatory notes

Candidates should be able to

(*a*) understand the concept of a function (and its notations) and the meaning of domain, codomain, range, and the equality of two functions;

(*b*) sketch the graphs of algebraic functions (including simple rational functions);

(*c*) use the six trigonometric functions for angles of any magnitude measured in degrees or radians;

(*d*) use the periodicity and symmetry of the sine, cosine, and tangent functions, and their graphs;

(*e*) use the functions e^x and $\ln x$, and their graphs;

(*f*) understand the terms one-one function, onto function, even function, odd function, periodic function, increasing function, and decreasing function;

(*g*) use the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{f}(x)|$;

(*h*) use the relationships between the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x + a)$, and $y = \mathrm{f}(ax)$;

(*i*) find composite and inverse functions and sketch their graphs;

(*j*) illustrate the relationship between the graphs of a one-one function and its inverse;

(*k*) sketch the graph of a piecewise-defined function;

(*l*) determine the existence and the value of the left-hand limit, right-hand limit, or limit of a function;

(*m*) determine the continuity of a function.

7. Differentiation

7.1 Derivative of a function

7.2 Rules for differentiation

7.3 Derivative of a function defined implicitly or parametrically

7.4 Applications of differentiation

Explanatory notes

Candidates should be able to

(*a*) understand the derivative of a function as the gradient of a tangent;

(*b*) obtain the derivative of a function from first principles;

(*c*) use the notations $\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$;

(*d*) use the derivatives of x^n (for any rational number n), e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$;

(*e*) carry out differentiation of $k\mathrm{f}(x)$, $\mathrm{f}(x) \pm \mathrm{g}(x)$, $\mathrm{f}(x)\mathrm{g}(x)$, $\frac{\mathrm{f}(x)}{\mathrm{g}(x)}$, $(\mathrm{f} \circ \mathrm{g})(x)$;

(*f*) find the first derivative of an implicit function;

(*g*) find the first derivative of a function defined parametrically;

(*h*) find the gradients of and the tangents and normals to the graph of a function;

(*i*) find the intervals where a function is increasing or decreasing;

(*j*) understand the relationship between the sign of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and concavity ;

(*k*) determine stationary points, local extremum points, and points of inflexion (end-points of an interval where a function is defined are not regarded as stationary or local extremum points);

(*l*) determine absolute minimum and maximum values;

(*m*) sketch graphs (excluding oblique asymptotes);

(*n*) find an approximate value for a root of a non-linear equation by using the Newton-Raphson method;

(*o*) solve problems concerning rates of change, minimum values, and maximum values.

8. Integration

8.1 Integral of a function

8.2 Integration techniques

8.3 Definite integrals

8.4 Applications of integration

Explanatory notes

Candidates should be able to

(*a*) understand indefinite integration as the reverse process of differentiation;

(*b*) use the integrals of x^n (for any rational number n), e^x, $\sin x$, $\cos x$, $\sec^2 x$;

(*c*) carry out integration of $k\mathrm{f}(x)$ and $\mathrm{f}(x) \pm \mathrm{g}(x)$;

(*d*) integrate a function in the form $\{\mathrm{f}(x)\}^r \mathrm{f}'(x)$, where r is a rational number;

(*e*) integrate a rational function by means of decomposition into partial fractions;

(*f*) use substitutions to obtain integrals;

(*g*) use integration by parts;

(*h*) evaluate a definite integral, including the approximate value by using the trapezium rule;

(*i*) calculate plane areas and volumes of revolution about one of the coordinate axes.

9. Linear programming

9.1 Problem formulation

9.2 Graphical solution

9.3 Simplex method

Explanatory notes

Candidates should be able to

(*a*) interpret a given problem and state the constraints;

(*b*) state the objective function that needs to be minimized or maximized;

(*c*) identify the feasible region;

(*d*) determine the optimal solution by considering values at the vertices of the feasible region;

(*e*) obtain the standard simplex form for a linear programming problem in standard form;

(*f*) construct simple simplex tableaux;

(*g*) determine the optimal solution by considering basic feasible solutions.

10. Network planning

10.1 Networks

10.2 Critical paths

10.3 Floats

Explanatory notes

Candidates should be able to

(*a*) understand the concepts of activities, events, and networks;

(*b*) construct and interpret networks;

(*c*) identify the critical activities and critical paths;

(*d*) calculate the minimum completion times;

(*e*) calculate and explain total, free, and independent floats.

11. Data description

11.1 Representation of data

11.2 Measures of location

11.3 Measures of dispersion

Explanatory notes

Candidates should be able to

(*a*) understand discrete, continuous, ungrouped, and grouped data;

(*b*) construct and interpret stemplots, boxplots, histograms, and cumulative frequency curves;

(*c*) derive and use the formula $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n(\bar{x})^2$;

(*d*) estimate graphically and calculate measures of location and measures of dispersion;

(*e*) interpret mode, median, mean, range, semi-interquartile range, and standard deviation;

(*f*) understand the symmetry and skewness in a data distribution.

12. Probability

12.1 Techniques of counting

12.2 Events and probabilities

12.3 Mutually exclusive events

12.4 Independent and conditional events

Explanatory notes

Candidates should be able to

(*a*) use counting rules for finite sets, including the inclusion-exclusion rule, for two or three sets;

(*b*) use the formulae for permutations and combinations;

(*c*) understand the concepts of sample spaces, events, and probabilities;

(*d*) understand the meaning of complementary and exhaustive events;

(*e*) calculate the probability of an event;

(*f*) understand the meaning of mutually exclusive events;

(*g*) use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

(*h*) understand the meaning of independent and conditional events;

(*i*) use the formula $P(A \cap B) = P(A) \times P(B \mid A)$.

13. Probability distributions

13.1 Random variables

13.2 Mathematical expectation

13.3 The binomial distribution

13.4 The normal distribution

Explanatory notes

Candidates should be able to

(*a*) understand the concepts of discrete and continuous random variables;

(*b*) construct a probability distribution table for a discrete random variable;

(*c*) understand the concept of the mathematical expectation;

(*d*) calculate the mean and variance of a discrete random variable;

(*e*) understand the binomial and normal distributions;

(*f*) use the probability function of the binomial distribution;

(*g*) standardise a normal variable;

(*h*) use normal distribution tables;

(*i*) use the binomial and normal distributions as models for solving problems.

14. Sampling and estimation

14.1 Random samples

14.2 Sampling distributions

14.3 Point estimates

14.4 Interval estimates

Explanatory notes

Candidates should be able to

(*a*) understand the distinction between a population and a random sample and between a parameter and a statistic;

(*b*) use the sampling distributions of the sample proportion and mean;

(*c*) use the central limit theorem;

(*d*) calculate unbiased estimates for the population proportion, mean, and variance;

(*e*) calculate and interpret the standard error of the estimator for the population proportion or mean;

(*f*) obtain and interpret a confidence interval for the population proportion;

(*g*) obtain and interpret a confidence interval for the population mean based on a large sample;

(*h*) obtain and interpret a confidence interval for the population mean based on a small sample from a normal distribution with known variance;

(*i*) determine the sample size in the estimation of the population proportion or mean.

15. Correlation and regression

15.1 Scatter diagrams

15.2 Pearson correlation coefficient

15.3 Linear regression lines

Explanatory notes

Candidates should be able to

(*a*) understand the concept of correlation and the relationship between the value of a correlation coefficient and the scatter diagram;

(*b*) calculate and interpret a Pearson correlation coefficient;

(*c*) understand the concept of linear regression;

(*d*) find the equation of a linear regression line by the method of least squares (transformation of variables may be required to obtain a linear relation);

(*e*) draw a regression line on the scatter diagram;

(*f*) use a suitable regression line for making predictions and understand the uncertainties of such predictions;

(*g*) use the relationships between correlation coefficient, regression coefficient, and coefficient of determination.

16. Time series and index numbers

16.1 Time series

16.2 Index numbers

Explanatory notes

Candidates should be able to

(*a*) understand the concepts of time series, including basic trend, seasonal variation, cyclical variation, and residual variation;

(*b*) understand the meaning of an additive model and a multiplicative model for a time series;

(*c*) smooth a time series by using moving averages;

(*d*) fit a trend line by the method of least squares;

(*e*) calculate seasonal variations;

(*f*) predict future values in a time series;

(*g*) understand and find index numbers;

(*h*) calculate and interpret Laspeyres and Paasche indices.

Form of Examination

The examination consists of two papers; the duration for each paper is 3 hours. Candidates are required to take both Paper 1 and Paper 2.

Paper 1 (same as Paper 1, Mathematics T) is based on topics 1 to 8 and Paper 2 is based on topics 9 to 16. Each paper contains 12 compulsory questions of variable mark allocations totalling 100 marks.

Reference Books

1. How, G. A., *Mathematics S: Pure Mathematics*, Pearson, 2003.
2. Ong, B. S., *Mathematics for STPM: Pure Mathematics*, Fajar Bakti, 2003.
3. Tey, K. S. & Tan, A. G., *STPM Mathematics: Mathematics S & Mathematics T – Paper 1*, Pelangi, 2003.
4. Bostock, L. & Chandler, S., *Core Maths for Advanced Level* (Third Edition), Nelson Thornes, 2000.
5. Smedley, R. & Wiseman, G., *Introducing Pure Mathematics* (Second Edition), Oxford, 2001.
6. Kaufmann, J. E. & Schwitters, K., *Algebra for College Students*, Brooks/Cole, 2000.
7. Stewart, J., *Calculus: Concepts and Contexts, Single Variable* (Second Edition), Brooks/Cole, 2001.
8. Chin, S. W., Khor, S. C., Leow, S. K., & Poh, A. H., *STPM Mathematics: Mathematics S – Paper 2*, Pelangi, 2003.
9. Soon, C. L., Tong, S. F., & Lau, T. K., *Mathematics S: Quantitative Method*, Pearson, 2003.
10. Bloomfield, I. & Stevens, J., *Complete Advanced Level Mathematics: Discrete & Decision*, Nelson Thornes, 2002.
11. Hebborn, J., *Heinemann Modular Mathematics for Edexcel AS and A-Level: Decision Mathematics 1*, Heinemann, 2000.
12. Crawshaw, J. & Chambers, J., *A Concise Course in Advanced Level Statistics* (Fourth Edition), Nelson Thornes, 2001.
13. Dickman, G., *Business Statistics* (Second Edition), Nelson Thomson Learning, 2001.
14. Johnson, R. A. & Bhattacharyya, G. K., *Statistics: Principles and Methods* (Fourth Edition), John Wiley, 2001.
15. Upton, G. & Cook, I., *Introducing Statistics* (Second Edition), Oxford, 2001.

SPECIMEN PAPER

950/1, 954/1 STPM

MATHEMATICS S

PAPER 1

MATHEMATICS T

PAPER 1

(Three hours)

MAJLIS PEPERIKSAAN MALAYSIA
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Instructions to candidates:

Answer **all** *questions.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae, and graph paper are provided.

1 By using the laws of set algebra, show that, for any sets A and B,

$$A \cap (A \cap B)' = A \cap B'. \qquad [3\ marks]$$

2 Solve the simultaneous equations

$$\log_4(xy) = \frac{1}{2},$$

$$(\log_2 x)(\log_2 y) = -2. \qquad [6\ marks]$$

3 Express $\dfrac{1}{(4r-3)(4r+1)}$ in partial fractions. Hence show that

$$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{1}{4}\left(1 - \frac{1}{4n+1}\right). \qquad [6\ marks]$$

4 Find the equation of the normal to the curve $x^2y + xy^2 = 12$ at the point (3, 1). [6 *marks*]

5 Evaluate the definite integral $\displaystyle\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx$. [6 *marks*]

6 Show that the mid-points of the parallel chords of the parabola $y^2 = 4ax$ with gradient 2 lie on a straight line parallel to the x-axis. [7 *marks*]

7 The functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x, \ x \in \mathbb{R};$$

$$\mathrm{g} : x \mapsto \cos x - |\cos x|, \ -\pi \le x \le \pi.$$

(i) Find the composite function $\mathrm{f} \circ \mathrm{g}$, and state its domain and range. [4 *marks*]

(ii) Show, by definition, that $\mathrm{f} \circ \mathrm{g}$ is an even function. [2 *marks*]

(iii) Sketch the graph of $\mathrm{f} \circ \mathrm{g}$. [2 *marks*]

8 Draw, on the same axes, the graphs of $y = e^{-\frac{1}{2}x}$ and $y = 4 - x^2$. State the integer which is nearest to the positive root of the equation

$$x^2 + e^{-\frac{1}{2}x} = 4. \qquad [3\ marks]$$

Find an approximation for this positive root by using the Newton-Raphson method until two successive iterations agree up to two decimal places; give your answer correct to two decimal places. [5 *marks*]

9 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 8 & 0 \\ 1 & 3 & 5 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} -2 & 0 & 0 \\ -1 & -5 & 0 \\ -1 & -3 & -2 \end{pmatrix}.$$

(i) Determine whether **A** and **B** commute. [3 *marks*]

(ii) Show that there exist numbers m and n such that $\mathbf{A} = m\mathbf{B} + n\mathbf{I}$, where **I** is the 3×3 identity matrix, and find the values of m and n. [6 *marks*]

10 Given that $y = \dfrac{1}{\sqrt{1+2x}+\sqrt{1+x}}$ where $x > -\dfrac{1}{2}$, show that, provided $x \neq 0$,

$$y = \frac{1}{x}\left(\sqrt{1+2x}-\sqrt{1+x}\right).$$ [3 *marks*]

Using the second form for y, express y as a series of ascending powers of x as far as the term in x^2. [6 *marks*]

Hence, by putting $x = \dfrac{1}{100}$, show that

$$\frac{10}{\sqrt{102}+\sqrt{101}} \approx \frac{79\,407}{160\,000}.$$ [3 *marks*]

11 Show that the curve $y = \dfrac{\ln x}{x}$ has a stationary point at $\left(\mathrm{e}, \dfrac{1}{\mathrm{e}}\right)$, and determine whether this point is a local minimum point or a local maximum point. [6 *marks*]

Sketch the curve. [3 *marks*]

Show that the area of the region bounded by the curve $y = \dfrac{\ln x}{x}$, the x-axis, and the line $x = \dfrac{1}{\mathrm{e}}$ is equal to the area of the region bounded by the curve $y = \dfrac{\ln x}{x}$, the x-axis, and the line $x = \mathrm{e}$.

[5 *marks*]

12 Show that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Deduce that both roots are real if $b^2 - 4ac \geq 0$ and are complex if $b^2 - 4ac < 0$. [4 *marks*]

Determine all real values of k for which the quadratic equation

$$x^2 - (k-3)x + k^2 + 2k + 5 = 0$$

has real roots. [5 *marks*]

If α and β are the roots of this quadratic equation, show that $\alpha^2 + \beta^2 = -(k+5)^2 + 24$. Hence find the maximum value for $\alpha^2 + \beta^2$. [6 *marks*]

SPECIMEN PAPER

950/2 STPM

MATHEMATICS S

PAPER 2

(Three hours)

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Instructions to candidates:

Answer **all** *questions.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae, and graph paper are provided.

1 Explain, with graphical illustrations, the relationship between the mean, median, and mode in a negatively skewed frequency distribution and in a positively skewed frequency distribution.
[3 *marks*]

2 If A and are two events with $P(A) = \frac{4}{7}$, $P(B) = \frac{2}{5}$, and $P[(A \cap B') \cup (A' \cap B)] = \frac{1}{3}$, find $P(A \cap B)$.
[3 *marks*]

3 A construction contractor orders sand, cement, and bricks from three different suppliers. The probabilities that the sand, cement, and bricks are sent to the project site on or before the agreed date are 0.80, 0.90, and 0.95 respectively. Find the probability that

(i) all supplies delivered to the project site are not late, [2 *marks*]

(ii) at most one kind of supply delivered to the project site is late. [3 *marks*]

4 A discrete random variable X can take only the values 1, 2, 3, and 4, with $P(X = 1) = P(X = 2)$ and $P(X = 3) = P(X = 4) = 2P(X = 1)$. If $A = \{1, 3\}$, find $P(A)$. [5 *marks*]

5 The marks of a test are normally distributed with mean 55 and standard deviation 12. A student who obtains 75 marks or more is given grade A, and a student who obtains 85 marks or more is awarded a certificate of excellence. Find the probability that

(i) a student receives grade A, [2 *marks*]

(ii) a student who receives grade A is also awarded a certificate of excellence. [4 *marks*]

6 The following table shows the quantities and prices of three electrical goods produced by a company from the year 1991 up to the year 1993.

Electrical Goods	Quantity (in thousand units)			Price (RM per unit)
	1991	1992	1993	
Microwave Oven	20	30	30	1000
Bread Toaster	10	20	30	100
Refrigerator	30	40	50	1500

By using the prices as weights and the year 1991 as the base year, calculate the Laspeyres quantity indices for the years 1992 and 1993. [3 *marks*]

Comment briefly on the difference between the rate of growth in production of the company for the year 1992 and that for the year 1993. [3 *marks*]

7 A carbonated drink company wishes to study the effect of television advertising on the sales of a brand of canned drink produced by the company in the Klang Valley. The frequency of broadcast each day of a week and the sales of the drink during the same week are given below.

Daily broadcast frequency	3	7	4	2	0	4	1
Sales of drink (in thousand cans)	11	18	9	4	7	6	3

Plot a scatter diagram and calculate the correlation coefficient for the above data. Comment on any relationship between the daily broadcast frequency and the sales of the drink during that week.
[8 *marks*]

8 The following table shows the lengths, in minutes, of 90 telephone calls made in a month by a trading company.

Length (minutes)	Frequency
0.0 - 2.9	12
3.0 - 5.9	19
6.0 - 8.9	35
9.0 - 11.9	13
12.0 - 14.9	8
15.0 - 20.9	3

(i) Draw a histogram for the above grouped data. [3 *marks*]

(ii) Calculate the median and the semi-interquartile range of the lengths of telephone calls made by the trading company. [7 *marks*]

9 A company produces three types of products A, B, and C. The production of these products requires two kinds of raw materials, M and N. The production engineer of the company has prepared the following table.

Raw material	Mass (g) required per unit of product		
	A	B	C
M	1	1	1
N	1	4	7
Profit per unit (RM)	2	3	1

The daily supplies of the raw materials M and N are limited to 3 kg and 9 kg respectively. Determine the linear programming model for the above production process. [4 *marks*]

By using the simplex method, find the maximum profit that the company can make in a day. [8 *marks*]

10 The following table shows the sales (in thousands) of a certain type of cars in Malaysia for each quarter from the year 1996 to the year 1999.

Year	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
1996	13	12	10	6
1997	18	16	16	9
1998	21	20	18	12
1999	25	26	23	18

(i) Find the centred four-quarter moving averages. [4 *marks*]

(ii) Plot, on the same axes, the original data and the moving averages. [4 *marks*]

(iii) Calculate the average seasonal variation for each quarter. [4 *marks*]

11 A company plans to manufacture a product *XYZ* which is a combination of two products *XY* and *YZ*. To carry out this plan successfully, the management needs to train the workers and get ready the raw materials for *XY* and *YZ*. Before *XY* and *YZ* are combined to produce *XYZ*, *YZ* is tested. The following table lists the activities and their durations taken to produce *XYZ*.

Activity	Description	Preceding activities	Duration (days)
A	Training the workers	-	6
B	Getting ready the materials	-	9
C	Preparing *XY*	*A*, *B*	8
D	Preparing *YZ*	*A*, *B*	7
E	Testing *YZ*	*D*	10
F	Combining *XY* and *YZ*	*C*, *E*	12

(i) Draw an activity network for the project. [3 *marks*]

(ii) Find the minimum time required to produce *XYZ*. [4 *marks*]

(iii) Calculate the total float for each activity and hence determine the critical path. [8 *marks*]

12 In a survey of 400 supermarkets throughout the country, it is found that 136 of them sell a daily essential product which contains a certain chemical exceeding the government-approved level.

(i) Estimate the percentage of supermarkets in the country which sell the product. [3 *marks*]

(ii) Obtain a 95% confidence interval for the percentage of supermarkets which sell the product. Give an interpretation of the confidence interval you obtain. [6 *marks*]

(iii) Determine the smallest sample size of supermarkets which should be surveyed so that there is a probability of 0.95 that the percentage of supermarkets which sell the product can be estimated with an error of less than 2%. [6 *marks*]